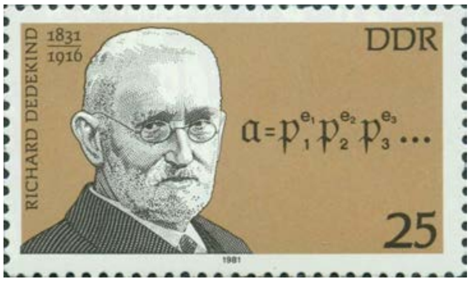
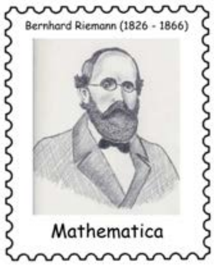
Richard Dedekind (October 6, 1831 – February 12, 1916)

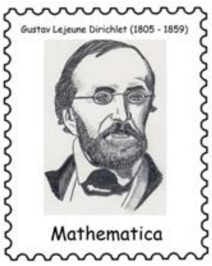
by Heinz Klaus Strick, Germany



The biography of Julius Wilhelm Richard Dedekind begins and ends in Braunschweig (Brunswick): The fourth child of a professor of law at the *Collegium Carolinum*, he attended the *Martino-Katherineum*, a traditional gymnasium (secondary school) in the city. At the age of 16, the boy, who was also a highly gifted musician, transferred to the *Collegium Carolinum*, an educational institution that would pave the way for him to enter the university after high school. There he prepared for future studies in mathematics.

In 1850, he went to the University at Göttingen, where he enthusiastically attended lectures on experimental physics by Wilhelm Weber, and where he met Carl Friedrich Gauss when he attended a lecture given by the great mathematician on the *method of least squares*. Gauss was nearing the end of his life and at the time was involved primarily in activities related to astronomy. After only four semesters, Dedekind had completed a doctoral dissertation on the theory of *Eulerian integrals*. He was Gauss's last doctoral student.





(drawings © Andreas Strick)

He then worked on his habilitation thesis, in parallel with Bernhard Riemann, who had also received his doctoral degree under Gauss's direction not long before. In 1854, after obtaining the *venia legendi* (official permission allowing those completing their habilitation to lecture), he gave lectures on probability theory and geometry.

Since the beginning of his stay in Göttingen, Dedekind had observed that the mathematics faculty, who at the time were mostly preparing students to become secondary-school teachers, had lost contact with current developments in mathematics; this in contrast to the University of Berlin, at which Peter Gustav Lejeune Dirichlet taught. On Gauss's death in 1855, Dirichlet was appointed his successor in Göttingen.

Dedekind worked closely with Dirichlet; eager to expand his mathematical horizons, he attended Dirichlet's lectures on such topics as number theory and the theory of partial differential equations.

In 1858, the *Polytechnikum* in Zurich (today ETH = Eidgenössische Technische Hochschule) advertised throughout Europe for professors of mathematics. Both Dedekind and Riemann applied. Dirichlet thought that Dedekind was better suited for the position, since he, in contrast to Riemann, had acquired experience as a lecturer on *elementary* topics for an *inexperienced audience*. Dedekind's lectures were distinguished, he wrote, *by their clarity, precision, and liveliness*.

A Swiss delegation visited both applicants and determined that Riemann was *too introverted to teach future engineers*. They therefore expressed a preference for Dedekind over Riemann.

In preparing his lectures in Zurich, DEDEKIND became aware that the arithmetic foundations of the differential calculus had never been adequately worked out. He therefore conceived of the idea to define the real numbers by what are today called *DEDEKIND cuts* in the set of rational numbers.

DEDEKIND soon, however, became unhappy with his position in Zurich. He had expected that the students would follow his lectures attentively and with close professional interest. Instead, he had to deal with the "childish behaviour" of some of them.

When in 1861, the *Collegium Carolinum* in Braunschweig was expanded into an institute of technology and advertised for a professor of mathematics, he applied for the position, in order to be able to return to his hometown, with the express request that he never again have to teach "lower mathematics". His conditions were accepted, and from then on, until his retirement in 1894, DEDEKIND worked in Braunschweig. He declined all offers, even from Göttingen.

In the early 1870s, the *Collegium Carolinum* was transformed into the *Herzogliche Technische Hochschule Carolo-Wilhelmina*. DEDEKIND was named its first director. When the duke of Braunschweig gave his consent for the new university, DEDEKIND took over direction of the building commission.

In 1872, DEDEKIND published a paper on *Continuity and Irrational Numbers* (Stetigkeit und Irrationale Zahlen), in which he presented his idea, developed earlier in Zurich, of *cuts*. (What DEDEKIND meant here by *continuity* is today known as *completeness*.)

The core of this idea is the following consideration: The number line is apparently complete, that is, without any holes. If one chooses some point, say *P*, then this point will correspond to either a rational or irrational number. Every point divides the number line into two parts:

- *All points of the line are decomposed into two classes in such a way that every point of the first class lies to the left of every point of the second class, and so there exists one and only one point that brings about this division of all points into two classes, this cutting of the line into two pieces.*

By this method, a rational number a divides the set $\mathbb{Q}$ of rational numbers into two subsets A_1 and A_2; all elements of the *lower class* A_1 are smaller than all elements of the *upper class* A_2.

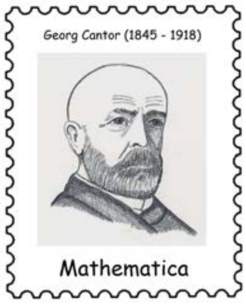
Regardless of whether one considers the rational number a as belonging to the *lower class* (as the largest number in A_1) or to the *upper class* (as the smallest number in A_2), the result is that the set $\mathbb{Q}$ of rational numbers is divided in two subsets by means of the rational number a.

Moreover, every irrational number b, for example $\sqrt{2}$, divides the set $\mathbb{Q}$ of rational numbers into two subsets A_1 and A_2; the elements of the *lower class* A_1 are all smaller than the irrational number b under consideration, and it, in turn, is smaller than all the elements of the *upper class* A_2: $A_1 = \{ x \in \mathbb{Q} \mid x^2 < 2\}$ and $A_2 = \{ x \in \mathbb{Q} \mid x^2 > 2\}$.

The *lower class* associated with an irrational number b, however, has no largest element, while the *upper class* has no smallest element. The irrational numbers thus fill the "holes" between all the pairs of subsets of rational numbers:

It is in this property, that not all cuts are brought about by rational numbers, that the incompleteness, or discontinuity, of the set of all rational numbers consists. Now, whenever one has a cut (A_1, A_2) not produced by a rational number, we have thereby created a new, irrational, number α, which we may consider to be completely defined by this cut (A_1, A_2) …

DEDEKIND also defined the order properties of the real numbers and their arithmetic operations by way of cuts.



Through a chance meeting during a vacation in 1874 with Georg Cantor, the creator of set theory, there arose a close friendship maintained through correspondence. Dedekind's contribution to the theory of infinite sets is the following theorem: A set is infinite if this set is similar to (in today's terminology, of the same cardinality as) a proper subset of itself. For example, the set of natural numbers $\mathbb{N}$ is of the same cardinality as a proper subset, e.g., the set of squares 1, 4, 9, 16, …, since the mapping $n \to n^2$ and the inverse mapping $n^2 \to n$ are bijections between the set of natural numbers $\mathbb{N}$ and the set of squares of natural numbers.

A result of Dedekind's work on the foundations of mathematics was the 1888 book *Was sind und was sollen die Zahlen?* (*The Nature and Meaning of Numbers*, or more literally, *What are the numbers and what are they for*?). In this work, he attempted to provide a foundation for the natural numbers through set-theoretic considerations. For him, *Numbers are … independent creations of the human imagination, which serve as a means of understanding the diversity of things more easily and more accurately*.

Giuseppe Peano employed these ideas by and large in his axiomatization of the natural numbers, which he published in 1889.

Dedekind was working on algebraic structures as early as during his time as a lecturer in Göttingen. He was one of the first mathematicians to make use of *Galois theory* in his lectures. In 1863, he had begun to edit Dirichlet's Lectures on *Number Theory*. In 1879, he extended those lectures with his own contribution *Über die Theorie der ganzen algebraischen Zahlen* (On the Theory of Algebraic Integers), which dealt with properties of *number fields*. For over one hundred years, mathematicians had been working on the question whether there existed additional mathematical objects that possessed some of the properties enjoyed by the set $\mathbb{N}$ of natural numbers, such as the fundamental theorem of arithmetic: Every natural number greater than 1 can be decomposed uniquely (up to the order of the factors) as a product of prime numbers.

Dedekind showed that an analogous theorem holds for so-called *rings of integers*:

For every proper ideal a there exists a decomposition into prime ideals p_1, p_2, p_3, … :
$a = p_1^{e_1} \cdot p_2^{e_2} \cdot p_3^{e_3} \cdot \ldots$ (see the reproduction of the DDR postage stamp).

Dedekind's enormous lifetime accomplishments, including his ground-breaking discoveries in algebra and number theory, led to numerous honours and awards from institutions both at home and abroad. When war broke out in 1914, Dedekind declined to sign a manifesto glorifying the aims of the war. The French *Academy of Sciences* thanked him for this in a special way: When Dedekind died, in 1916, the *Academy* was the first institution – and this despite being at war with Germany – to publish an obituary honouring him.

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https://www.spektrum.de/wissen/julius-wilhelm-richard-dedekind-1831-1916/1124386

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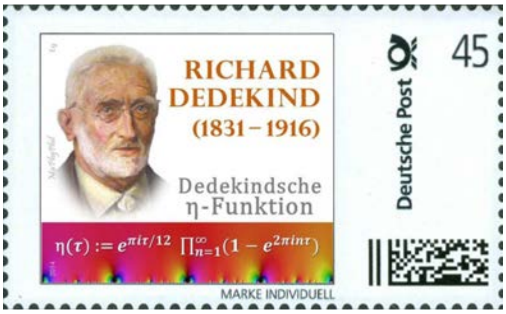

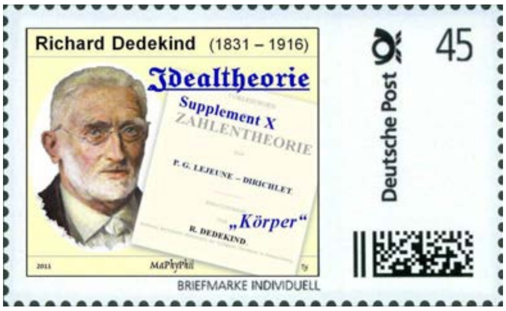

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