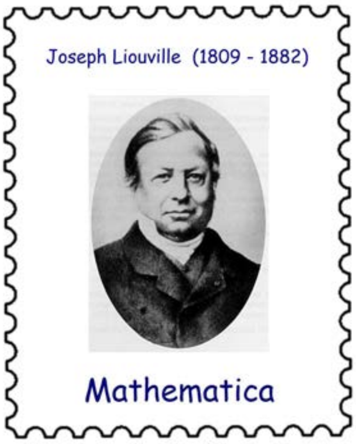
JOSEPH LIOUVILLE (March 24, 1809 – September 8, 1882)

by HEINZ KLAUS STRICK, Germany



In function theory, that is the differential and integral calculus for functions defined on the set of complex numbers, the following theorem is central:

LIOUVILLE's Theorem

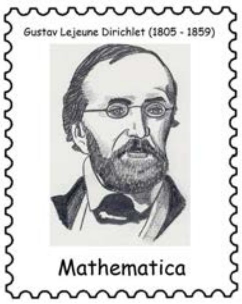
If an everywhere differentiable function f of a complex variable is bounded, i.e. if there is a positive real number *c* so that: $|f(z)| < c$ for all $z \in \mathbb{C}$, then the function f is constant.

This remarkable theorem does not apply to real-valued functions, as can be seen in the example of the sine or the cosine function, which are differentiable everywhere on the set of real numbers and are bounded. The theorem named after LIOUVILLE was proved by CAUCHY in 1844.

JOSEPH LIOUVILLE grew up with an uncle during the first years of his life, as his father, an officer in the Napoleonic army, took part in the campaigns throughout Europe. After his father's happy return home, the family was reunited. JOSEPH LIOUVILLE attended school in Toul (near Nancy) – with success, and he was allowed to continue his education at the prestigious *Collège St. Louis* in Paris.

In 1825 he was admitted to study at the *École Polytechnique*, attended lectures on analysis and mechanics by ANDRÉ-MARIE AMPÈRE and FRANÇOIS ARAGO and passed his exams in 1827.

He then continued his studies at the *École des Ponts et Chaussées*, but discovered during practical work in the field that this caused him health problems. As he had in the meantime written a number of scientific papers, he hoped to find a permanent position at a university. In order to earn a living, he also took on teaching at various schools – up to 40 hours a week, sometimes at the expense of quality – and he also often talked over the heads of his students.



The fact that LIOUVILLE – because of the low reputation of French journals – had to publish his articles abroad, especially in CRELLE's Journal, annoyed him, so in 1836 he decided to publish a journal himself. He kept his *Journal de Mathématiques Pures et Appliquées* until 1874 and, together with his friend PETER GUSTAV LEJEUNE DIRICHLET, he made his mark on the journal with his comments and additions, so that after a short time it was generally known as the *Journal de LIOUVILLE*.

After an unsuccessful application for a vacant professorship at the *École Polytechnique*, LIOUVILLE was accepted as a lecturer at the *Collège de France* in 1837 and when another position at the *École Polytechnique* became available in 1838, his application was successful.

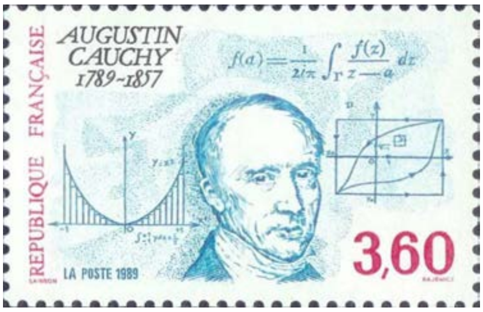
Now he could allow himself to organise the course for himself. He spent the summer in Toul, working on the next issue of his journal and dealt with research topics of his own choice and then from November to July he taught in Paris.

In 1843 he also tried to get a permanent professorship at the *Collège de France*. In the application process he was subject to a hated competitor, the self-confident Count Guglielmo Libri Carucci Dalla Sommaja, who had drawn attention to himself through publications on the history of mathematics in Italy. Liouville felt humiliated and gave up his teaching position at the *Collège*.

From 1842 onwards, Liouville studied the writings of Évariste Galois, which had not yet been published. He recognised the importance of Galois' theories and, in 1843, at the meeting of the *Académie de France*, of which he had been a member since 1839, he announced that he would soon publish an annotated version in his journal. However, this did not happen until three years later and - despite announcements to the contrary - without any comments or additions. (It is known from his students Bertrand and Hermite that Liouville lectured on these topics in his lectures).

During the revolution of 1848, he was elected to the *Constituent Assembly* at Arago's request and took part in the deliberations within the faction of moderate republicans. When his renewed candidacy failed in the elections the following year, he withdrew bitterly from the political bodies.

Overall, Liouville went through an extremely depressive period which ended abruptly when he was appointed to succeed Libri as professor at the *Collège de France* in 1851.

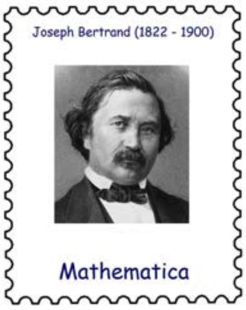


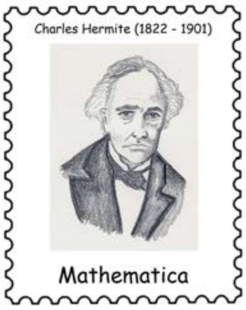
Libri had fled to England in 1848; not for political reasons, but because he was convicted of the theft of books (some 30,000 volumes in all, including very valuable copies), which Libri took with him when he fled. (Because of his supposed qualifications, he had been commissioned by the government to make an inventory of the books confiscated from the nobility during the French Revolution.) The fact that he was even able to beat Augustin Cauchy in his application lifted Liouville 's spirits even more.

The following years can be called Liouville's most productive phase. He published numerous research results, but these suffered from a problem. By taking on too many lectures, he did not manage to optimise his results, nor did he succeed in formulating the proofs of the mathematical theorems he had discovered in such a way that they always met his own high standards.

When Jacques Charles François Sturm, with whom he had also worked successfully (*Sturm-Liouville's differential equations*) died in 1857, he also took over his mechanics lectures at the *Sorbonne* in Paris. With the death of his friend Dirichlet, with whom he had maintained extensive correspondence, a particularly intensive creative phase came to an end in 1859.

His activity as editor of a journal (58 volumes in total) was always connected with a careful review of the contributions, and it encouraged him to make valuable additions.





The promotion of young authors (such as Joseph Bertrand and Charles Hermite) in their first publications was of particular concern to him. Although he himself did not have the time to write a book, he published, among others, a work by Gaspard Monge: *Application de l'analyse à la géométrie* (Application of Analysis to Geometry, 1850), with his own contributions on the differential geometry of curves and surfaces in space.

Niels Henrik Abel and Galois had shown that there can be no general solution method for equations of higher than 4th degree; Liouville was able to prove in 1840 that the Eulerian number *e* cannot be the solution of a 2nd or 4th degree algebraic equation. In 1844, he succeeded in proving the existence of transcendental numbers (i.e. real numbers which are not the solution of any algebraic equation with rational coefficients) by constructing whole classes of such numbers and representing them by means of continued fractions. He also proved that the Liouville number *L*, named after him, is transcendental:

$$L = 10^{-1} + 10^{-2} + 10^{-6} + 10^{-24} + \ldots = \sum_{k=1}^{\infty} 10^{-k!} = 0{,}1|1|0001|0000000000\,00000001|\ldots$$

Liouville wrote a total of about 400 papers, half of them on number theory and a quarter on analysis and its applications in physics. Numerous terms still remind us today of his outstanding achievements (*Liouville 's surface*, Liouville 's formulas for geodesic curvature, *Liouville 's normal form of differential equations*) and prove his versatility.

In the 1830s he had already investigated which functions can be called *elementary* (*Liouville's functions*) and proved that the functions $f(x) = \frac{\sin(x)}{x}$ and $f(x) = e^{-x^2}$ are not elementary because they cannot be integrated in an elementary way (i.e. one cannot specify an explicit antiderivative for these functions).

Until his death, he regularly attended the meetings of the *Paris Academy*, taking over its chairmanship in 1870 and he was also involved in the *Bureau des Longitudes*, of which he had been a member since 1840 (succeeding Siméon Denis Poisson).

Liouville was honoured by the *Royal Society*, the *Academies* of Göttingen and Sweden, and died in Paris at the age of 73.

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