**George Pólya** (December 13, 1887 – September 7, 1985)

by Heinz Klaus Strick, Germany


(drawings © Andreas Strick)

George Pólya grew up in Budapest as the fourth child of Anna Deutsch and Jakab Pólya. The father had taken the family name György only five years before the birth of his son, because this name sounded more Hungarian and less Jewish than the original name Pollák. The fact that the parents converted to the Catholic faith with their three children in 1886 and that György was also baptised a Catholic soon after birth was helpful with regard to the father's intended professional career.

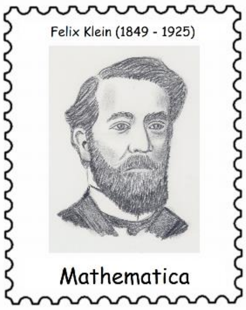
But in 1897, a few weeks after Jakab Pólya was finally able to take up a position as a private lecturer in economics and statistics at the University of Budapest, he died. George's two older sisters found employment in the insurance company where their father had worked as a lawyer before becoming a lecturer.

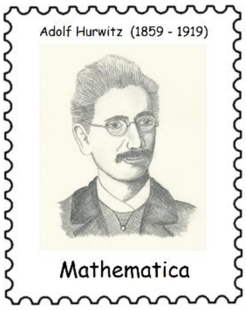
After primary school, George Pólya attended a grammar school where the classical languages Latin and Greek were just as important as Hungarian and German. Biology and literature were the subjects in which he achieved top marks.

There was no evidence of a special mathematical talent during his school years. Looking back on his school days, he attributed the fact that he did not develop any particular interest in mathematics at that time to the incompetence of his mathematics teachers.

At the age of 18 – at the insistence of his mother, who hoped for a family continuation of his father's career – he began studying law, but he dropped this after only one semester because it bored him. He then studied languages for two years and obtained authorisation to teach Latin and Hungarian at the grammar school. His interest then turned to philosophy, but his professor advised him to study mathematics and physics first so that he would be better prepared to study philosophy. And so he attended the lectures of Lipót Fejér and Loránd Eötvös ...





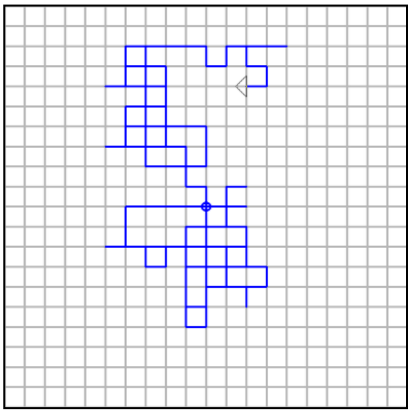

In 1910 he spent an academic year at the University of Vienna, and in 1911 Pólya received his doctorate in Budapest with a thesis on geometric probabilities. In 1912 and 1913 he spent some time in Göttingen and met Constantin Carathéodory and Felix Klein and he then went to Paris.

In 1914 the University of Frankfurt offered him a position, which he initially accepted, but then he decided to work at the ETH Zurich, as he hoped to gain a lot from working with Adolf Hurwitz. When Hurwitz died unexpectedly in 1919, he took care of his scientific estate.

At the outbreak of the World War, he was initially in no danger of being called up because of an old sports injury. However, when the situation of the Hungarian army worsened, he was also asked to return home. He refused and had himself naturalised in Switzerland. Because of the danger of subsequent punishment, however, he did not dare to travel to Hungary again until 1967.

Among his enthusiastic listeners was János Neumann (John von Neumann) the only student – as Pólya later confessed – of whom even he, the lecturer admired by all the students, had been "afraid". Hardly a lecture had gone by in which he had formulated a problem for which his "student" had not been able to present a solution at the end of the lecture.


Pólya caused a sensation in 1921 with his research results on random walks in $n$-dimensional coordinate grids:



An object wanders from the origin in single steps of length 1 and then rotates randomly and with equal probability in one of the $2n$ possible directions. Pólya could prove that for every random walk starting at the origin (the term goes back to Karl Pearson) on the 1-dimensional axis and in the 2-dimensional plane, the following is true:

> *At some point, the random walk reaches every grid point of the axis or the plane, i.e. with probability 1.*

However, this no longer applies for $n \geq 3$.

His investigations on ornamental groups of the plane in 1924 were also impressive. Pólya proved that there are exactly 17 so-called *crystallographic groups* of periodic patterns (tessellations) of the plane. Through the geometric elements belonging to the groups, patterns are mapped back onto themselves.

*(Source: https://en.wikipedia.org/wiki/Wallpaper_group)*

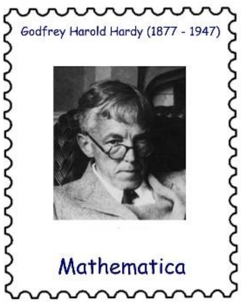
Examples of the geometric transformations are:


Translations (displacements), axis reflection, rotations by 180° (reflection in a point), rotations by 120°, by 90° or by 60° and combinations of these operations (such as *glide reflections*).

From the fact that 17 such groups exist, it follows that 93 different tessellations of the plane are possible.

Maurits Cornelis Escher translated Pólya's results into graphic works.

*(source: http://www.mathe.tu-freiberg.de/~hebisch/cafe/algebra/ornamentgruppen.html)*




As the first foreign scholar of the Rockefeller Foundation, Polya was able to conduct research at the Universities of Oxford and Cambridge in 1924. His collaboration with Godfrey Harold Hardy and John Endensor Littlewood resulted in a joint book on *Inequalities*, which appeared in 1934.

In 1925, together with Gábor Szegő, whom he still knew from his student days at the University of Budapest and who in the meantime had a position as a private lecturer at the University of Berlin, Pólya published two volumes with the title *Aufgaben und Lehrsätze der Analysis* (Problems and theorems from analysis) – a masterpiece that deviated from the type of analysis books published so far.

The special feature of these volumes, which were later republished several times, was that they were not arranged according to topics, but according to the method of proof.

PÓLYA later explained how he came up with this unusual approach:

The approach of the mathematicians was conclusive for him, the late-comer, but he kept asking himself:

> *How did mathematicians come up with the idea of discovering the statements of these theorems in the first place?*

His work received its first recognition in 1928 when his position at the ETH was converted into a full professorship. In 1933, PÓLYA again received a scholarship from the *ROCKEFELLER Foundation* and spent a few months in Princeton and Stanford. Although he then returned to Zurich, the political situation in Europe prompted him to emigrate to the USA in 1940, where he was appointed to a chair at Stanford University in 1942. Even after his retirement in 1953, he lectured as emeritus professor (and continued in good health) until he was 90 years old. Through his work, PÓLYA gave numerous impulses for the further development of theories in different areas of mathematics.

In 1918, for example, he wrote articles on series, number theory, combinatorics and voting systems, and the following year he contributed to astronomy and probability theory. In the years 1926 to 1928 alone, he published 31 scientific papers that received international recognition.

The total of more than 250 publications by the ingenious mathematician provided inspiration for further research in complex analysis, mathematical physics, combinatorics and probability theory (he coined the term *Central Limit Theorem*) as well as in geometry. This makes him probably one of the most influential mathematicians of the 20th century.

Since his time in Zurich, PÓLYA had been concerned with the question of how mathematics should be taught. The study of *heuristics* ("*logic of invention*") and the *logic of plausible reasoning* should be the basis of the training of all mathematics teachers.

At first, he did not find a publisher for his work *Schule des Denkens* (School of Thinking), which was written in German. The English version *How to solve it* appeared in Princeton in 1945. After the importance of this work was recognised, it was translated into 20 languages and sold more than a million copies in total.

In the preface PÓLYA pointed out:

> *If* (... *the mathematics teacher* ...) *fills the time at his disposal with drilling his pupils in practised procedures, he diminishes their interest and hinders their intellectual development; then he makes poor use of his opportunity. But if he awakens his pupils' thirst for knowledge by setting them tasks adapted to their knowledge and helps them to solve the tasks by skilful questions, he will develop in them a taste for independent thinking and show them ways to do so.*

In the book, he described a 4-step procedure for solving a task:

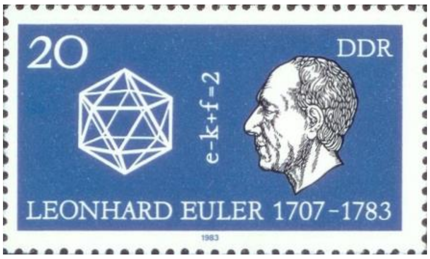
> *understanding the task – making a plan to solve it – carrying out the plan – looking back*.

In a small *Dictionary of heuristics*, PÓLYA mentioned various strategies such as *generalising* and *specialising*, *decomposing into sub-problems*, *looking for analogies*, *working forwards* and *backwards* – all procedures explained by convincing examples.

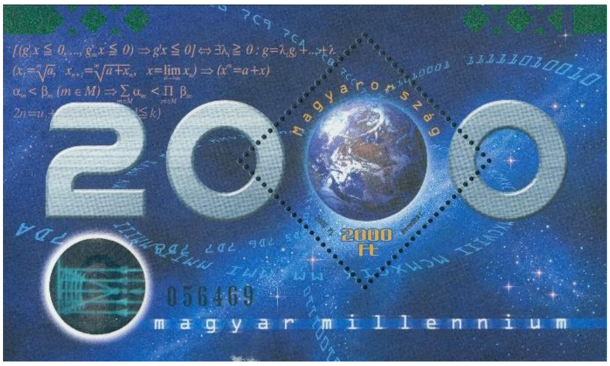
In particular, he repeated again and again the encouraging advice:

> *If you can't solve a problem, then there is an easier problem you can solve: find it.*

After numerous positive responses, *Mathematics and Plausible Reasoning* and *Mathematical Discovery* followed in the 1950s and 1960s, in which he clarified his proposals with further examples, among other things by explaining how LEONHARD EULER had proceeded with some of his discoveries.




Even if PÓLYA's approaches are viewed somewhat less enthusiastically today than they were a few decades ago, it can be said that the history of *problem solving* in mathematics education can be divided into two eras: the time *before* PÓLYA and the time *after* PÓLYA.



*If you illuminate the millennium block of the Hungarian post office with UV light, you can see the names of 57 famous Hungarian mathematicians, including PÓLYA GYÖRGY, FEJÉR LIPÓT, SZEGŐ GÁBOR, NEUMANN JÁNOS (in Hungarian, the first names are placed afterwards).*

First published 2015 by Spektrum der Wissenschaft Verlagsgesellschaft Heidelberg

https://www.spektrum.de/wissen/george-polya-ungarisch-amerikanischer-mathematiker/1355897

Translated 2022 by John O'Connor, University of St Andrews

Here are some individual (not offially issued) stamps about LEONARD EULER: