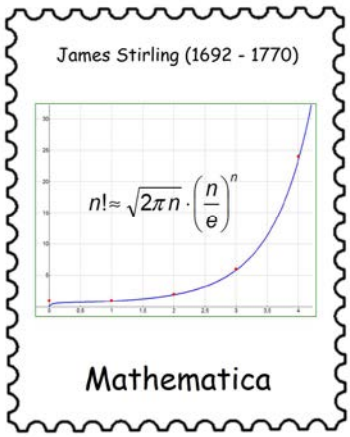
JAMES STIRLING (May 1692 – December 2, 1770)



by HEINZ KLAUS STRICK, Germany

One of those mathematicians who would have long been forgotten if their name were not repeatedly quoted in connection with a formula or a technical term is the Scot JAMES STIRLING.
No portrait of the scientist has survived. Little is known about the first years of his life either.

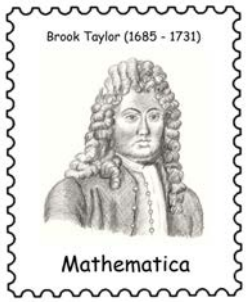
His father ARCHIBALD belonged to the *Jacobites*, the followers of the Catholic King JAMES II of the House of Stuart (JACOMUS or JACOB in Latin), who was expelled from the English throne in 1688 and living in French exile.

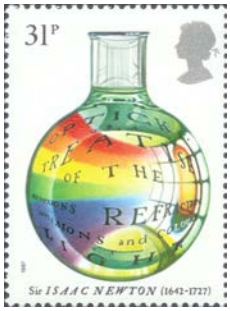
After the dissolution of the Scottish Parliament and the incorporation of Scotland into the Kingdom of England and Wales in 1707 (*Act of Union*), resistance groups formed. In the wake of a failed invasion by the French *pretender*, there was a wave of arrests among Scottish supporters. ARCHIBALD STIRLING was charged with high treason, but acquitted after a trial.

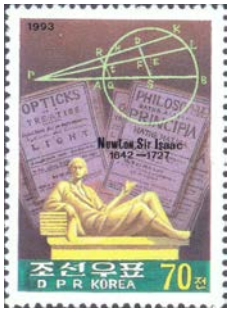
It is not known what schooling JAMES STIRLING had, and it is not clear whether he attended the University of Glasgow. The first document mentioning him is his enrolment at *Balliol College*, Oxford in January 1711 – an educational institution for Scottish students whose graduates usually returned home as *Church of England* priests.

After the first great *Jacobite revolt* in 1715 (*Battle of Sheriffmuir*), the student JAMES STIRLING was supposed to take an oath to the British crown. Since he refused, he lost his scholarship and had to leave the university without a degree.

Still, he appears to have stayed in the area for a while, as his name appears on an attendance list at a *Royal Society* event in 1717 when BROOK TAYLOR lectured on solving equations and logarithms. And in the same year STIRLING published his first work.





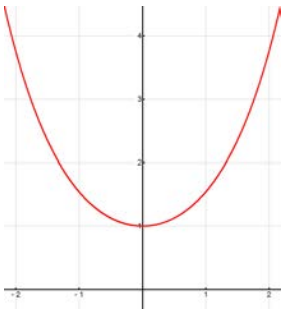


(drawing: © Andreas Strick)

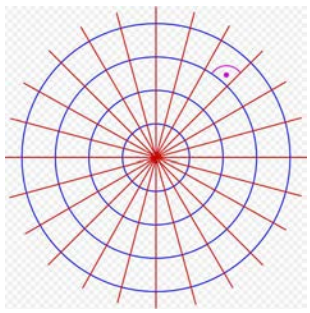
In the treatise with the title *Lineae Tertii Ordinis Neutonianae* (NEWTON's third order curves), STIRLING dealt with NEWTON's 1704 *Enumeratio Linearum Tertii Ordinis* (An enumeraton of cubic curves), in which he distinguished 72 types of plane curves of the 3rd order (as an appendix to *Opticks*). It is noteworthy that, as an ex-student without a degree and despite NEWTON's systematic approach, STIRLING was able to add another four curve types to his list.

In addition, the first publication contains STIRLING's remarks on the problem of the *brachistochrone* (the curve on which a frictionless mass slides down fastest between two points), on the *catenary* (the curve obtained by suspending a chain at both ends) and the problem of *orthogonal trajectories* (how to find a set of curves that intersect a given set of curves perpendicularly).

Brachistochrone
(Wikipedia)



Catenary

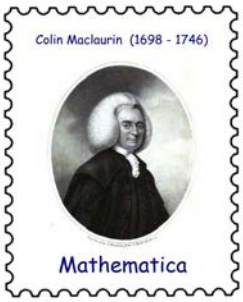


Concentric circles with orthogonal trajectories
(Wikipedia)

STIRLING sent a copy of the book to ISAAC NEWTON - in the hope of his recognition and future support. In fact, this first work was not dedicated to the esteemed NEWTON, but to another member of the *Royal Society*, namely NICHOLAS TRON, Ambassador of the Republic of Venice, who had promised STIRLING a chair of mathematics in Italy. STIRLING therefore accompanied the ambassador when he returned home in the summer of 1717. On arrival, however, STIRLING discovered that there was no job for him.

Despite the setback, he stayed in Italy and continued to do mathematics. From Venice he sent his second work *Methodus differentialis Newtoniana illustrata* (NEWTON's differential method illustrated) to the *Royal Society* in London in 1719.

In Padua, which belonged to the Republic of Venice, he befriended NICOLAUS BERNOULLI, who in 1713 had published *Ars conjectandi* (The art of conjecturing) from the works of his uncle JACOB BERNOULLI and in 1716 took over the GALILEO chair at the University of Padua.



This friendship encouraged STIRLING to write a letter to NEWTON, in which he offered himself as a mediator in the dispute between the supporters of NEWTON and those of LEIBNIZ and the BERNOULLIS.

In 1722 STIRLING returned to Glasgow. There are indications that this happened under dramatic circumstances: allegedly he had found out the secrets of the Venetian glassmakers and fearing assassination, he fled the country.

From 1724 he got a job, with the help of the Scottish mathematician COLIN MACLAURIN and with the support of NEWTON, as a teacher at a prestigious London school. He had to pay for the necessary aids, for example his drawing equipment. In 1726 he was accepted as a member of the *Royal Society*.

In 1730, STIRLING's most important mathematical work, *Methodus Differentialis: sive Tractatus de Summatione et Interpolatione Serierum Infinitarum*, was published, which he dedicated to NEWTON.

As the title suggests, it deals with the limits of infinite series and with the determination of values by interpolation.

With great virtuosity, STIRLING examined the question of how the convergence of these series can be "accelerated" through the transformation of the series formulas. With the help of his chosen approaches, he succeeded in estimating the limits of infinite series which were not known previously.

Thus he calculated the infinite sum of the reciprocal square numbers $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+...$ to be 1.644934065. This deviates only in the 9th decimal place from the actual limit value $\frac{\pi^2}{6}$ which LEONHARD EULER deduced in 1735, five years later.

And for the LEIBNIZ series $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+-...$ which was known to be slow to converge to its limit value of $\frac{\pi}{4}$ he found the value 0.7853981634 with an accuracy that could hardly be obtained by the continuous addition or subtraction of fractions.

For the factorial sequence $T_n = n!$ defined by: $T_{n+1} = (n+1) \cdot T_n$ he used clever interpolation to determine $T_{1.5} \approx 1.7724538506$ to ten digits which agreed very well with the actual value of $\sqrt{\pi}$ (proved in 1749 by EULER, in connection with the *Gamma function*).

The best-known theorem of the work is the approximation formula

$$n! \approx \sqrt{2\pi \cdot n} \cdot \left(\frac{n}{e}\right)^n .$$

He discovered this, at about the same time as ABRAHAM DE MOIVRE, when he was looking for a method to determine logarithms of factorials.

In 1735 he wrote *Of the figure of the Earth, and the variation of gravity on the surface*, in which he supported NEWTON's theory of the flattening of the earth at the poles. In the following years, he followed the results of the expeditions of DE LA CONDAMINE and MAUPERTUIS to Peru and Lapland with excitement.

In 1735 STIRLING took over the post of managing director of the *Scotch mining company* in Lanarkshire in Scotland and he was completely absorbed in this work. Among other things, he wrote a paper on improving ventilation in mines.

He also dealt with the construction of locks on the River Clyde, so that Glasgow could be reached by larger ships - this was the basis for the future prosperity of this city. STIRLING hardly had any time left for mathematical studies. It took him almost two years to reply to a letter from LEONHARD EULER, in which he was enthusiastic about STIRLING's *Methodus Differentialis* and presented a wealth of ideas.

Nevertheless, EULER suggested STIRLING for admission to the *Berlin Academy of Sciences* and the election took place in 1746.

In 1745 there was a final uprising of the Jacobites and the heir to the throne CHARLES EDWARD of the House of Stuart approached Edinburgh with an army. MACLAURIN, professor of mathematics at the university, was fatally wounded as a result of the war. STIRLING was proposed as his successor, but not appointed because of his sympathy for the Jacobites.

In 1753 STIRLING gave up his membership in the *Royal Society* as he could no longer afford to pay his subscription.

For the rest of his life he concentrated on the care of the mining company he led and after his death this office was passed on to his son-in-law.

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https://www.spektrum.de/wissen/james-stirling-1692-1770-der-mann-mit-der-fakultaetenformel/1376415

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