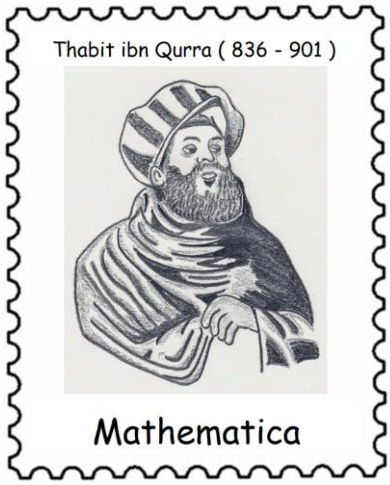
THABIT IBN QURRA (836 – 901)

by HEINZ KLAUS STRICK, Germany



Around 825, the Abbasid ruler AL-MA'MUN (a son of HARUN AL-RASCHID) founded the *House of Wisdom* in Baghdad to collect all available scientific writings - as was once done in the library in Alexandria. Works written in foreign languages were to be translated into Arabic. It was no coincidence that such a centre was established there: via Samarkand, the Chinese invention of papermaking had become known in the Islamic lands and in 795, the first paper mill was built in Baghdad.

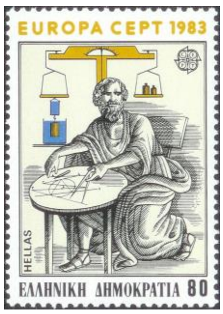
(drawing © Andreas Strick)

The caliph had the writings of antiquity researched throughout the empire; he also sent legations to the rulers of neighbouring empires, e.g. to Byzantium, in order to at least obtain copies of important books. The wealthy brothers ABU JAFAR MUHAMMED, AHMED and AL-HASAN IBN MUSA IBN SHAKIR participated both in the search for unknown writings and in the translations.

On one of his journeys, MUHAMMED met the articulate money-changer AL-SABI THABIT IBN QURRA AL-HARRANI in Harran (Mesopotamia), who spoke fluent Arabic and Greek in addition to his mother tongue, Syriac (a language derived from Aramaic). As the name AL-SABI indicates, THABIT belonged to the Sabian religious community, which was tolerated by Islam and worshipped the moon, the planets and the stars as deities.

Since some of the discovered writings were no longer preserved in the original Greek, but still existed in Syriac (translated by learned Nestorians, members of the Assyrian Church of the East), THABIT seemed particularly well suited as a collaborator; and so MUHAMMED IBN MUSA persuaded him to come with him to Baghdad.

Working on scholarly texts was not an easy activity; one could not be sure how reliable the person who made the copy at some point was, but equally how authentic the writing was from which that copy was made. If a translator had several copies of the same treatise, then he had to compare the representations and, if there were discrepancies, find out which version was probably the original. It seems that THABIT IBN QURRA had more than fulfilled the hopes placed in him.



However, the BANU MUSA brothers also knew how to arouse his interest in mathematics, and he proved to be an erudite, highly gifted student. Today, THABIT IBN QURRA is considered the most important mathematician of the 9th century.

Initially, THABIT translated the *Elements* of EUCLID as well as some writings of ARCHIMEDES: *On the Sphere and the Cylinder, Measuring the Circle, On the Division of the Circle into Seven Equal Parts, The Lemmata*.

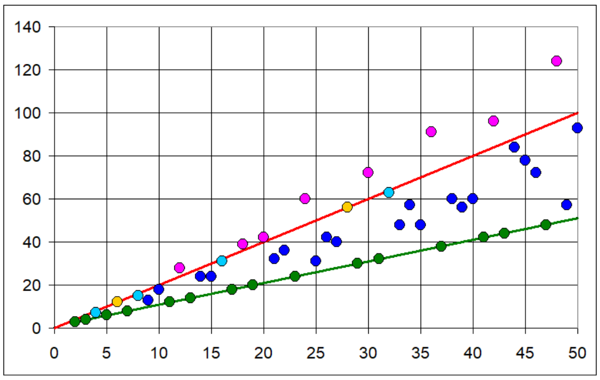
The first four of the still existing seven books of the *Conica* of APOLLONIUS were translated by AHMED IBN MUSA, and then the three others by THABIT.

But he was not content with translating the ancient works; his critical analysis led to new mathematical insights. In total, he wrote more than 70 works of his own on various topics of mathematics, and also on astronomy and astrology, physics and music, philosophy and medicine. He was also a successful physician and astronomer (astrologer). His son SINAN IBN THABIT and his grandson IBRAHIM IBN SINAN IBN THABIT continued the legacy of their father and grandfather as distinguished scholars.

Many of his translations and works were translated into Latin in the 12th century in Toledo, the centre of Islamic science in Europe, by, among others GERHARD OF CREMONA, including THABIT's extended treatise of the mechanics of ARCHIMEDES – and so the circle closed: the writings of antiquity returned to the consciousness of the newly awakening European science.

In the *Elements*, EUCLID distinguished between *abundant, deficient* and *perfect* natural numbers – these are numbers *n* for which the sum σ(*n*) of the proper integer divisors is greater or smaller than *n* or equal to *n*.

For example,
- the number 12 is abundant (because σ(12) = 1+2+3+4+6 > 12),
- the number 10 is deficient (because σ(10) = 1+2+5 < 10) and
- the number 28 is perfect (because σ(28) = 1+2+4+7+14 = 28).



green dots / green line: prime numbers
blue dots: other deficient numbers
light blue dots: powers of 2
golden dots / red line: perfect numbers
purple dots: abundant numbers

PYTHAGORAS already knew (at least) one pair (*a, b*) with σ(*a*) = *b* and σ(*b*) = *a*, namely (220; 284). Such numbers are called *friendly* numbers.

THABIT proved the following theorem:

- If the natural numbers $p_1 = 3 \cdot 2^{n-1} - 1$, $p_2 = 3 \cdot 2^n - 1$ and $p_3 = 9 \cdot 2^{2n-1} - 1$ are prime numbers, then $a = p_1 \cdot p_2 \cdot 2^n$ and $b = p_3 \cdot 2^n$ are friendly numbers.

However, the THABIT rule does not yield such a pair for all *n*.

For n = 2, we get $p_1 = 3\cdot 2^1 - 1 = 5$, $p_2 = 3\cdot 2^2 - 1 = 11$ and $p_3 = 9\cdot 2^3 - 1 = 71$ and thus $a = 5\cdot 11\cdot 2^2 = 220$ and $b = 71\cdot 2^2 = 284$.

But for n = 3, 5 and 6, one of the numbers p_1, p_2, p_3 is not a prime number; the prerequisites of the theorem are therefore not fulfilled.

For n = 4 one gets $p_1 = 3\cdot 2^3 - 1 = 23$, $p_2 = 3\cdot 2^4 - 1 = 47$ and $p_3 = 9\cdot 2^7 - 1 = 1151$ and thus $a = 23\cdot 47\cdot 2^4 = 17296$ and $b = 1151\cdot 2^4 = 18416$.

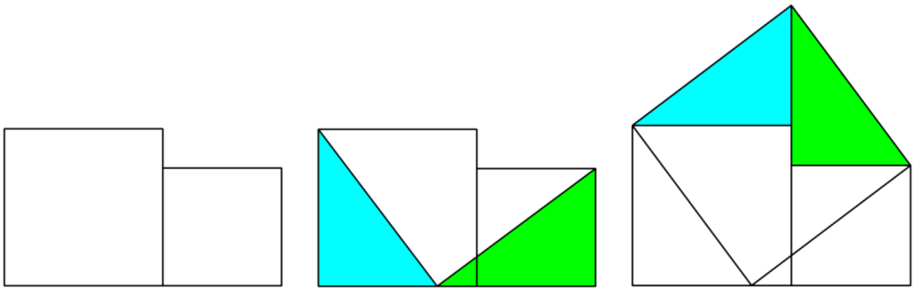
For n = 7 one gets $p_1 = 3\cdot 2^6 - 1 = 191$, $p_2 = 3\cdot 2^7 - 1 = 383$ and $p_3 = 9\cdot 2^{13} - 1 = 73727$ and thus $a = 191\cdot 383\cdot 2^7 = 9363584$ and $b = 73727\cdot 2^7 = 9437056$.

Whether THABIT examined the pairs in question is not known.

IBN AL BANNA mentioned the pair (17296, 18416) in the 14th century, and these were rediscovered in 1636 by the 19-year-old PIERRE DE FERMAT.

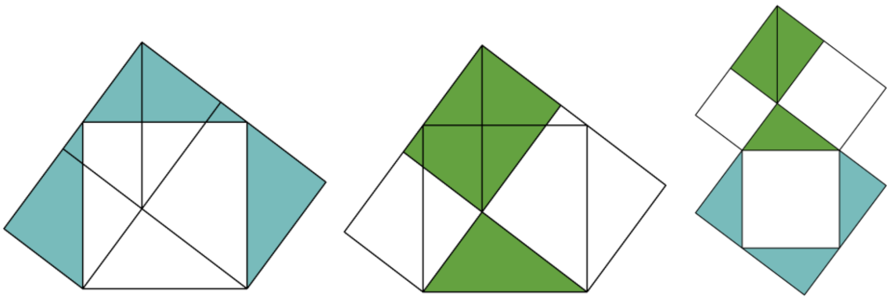
From THABIT also came three (new) proofs of the theorem of PYTHAGORAS.

1st proof:

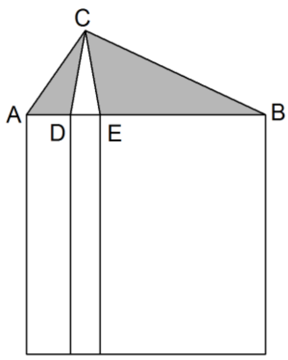


(graphics from: *Mathematics is beautiful*, Springer, 2021)

The ingenuity of the approach to the 2nd proof is particularly evident if one colours the figure differently or if one turns a part of the figure by 180° (see reduced figure on the right).



The 3rd proof (cf. figure on the right) deals with the generalisation of the PYTHAGOREan theorem:



In a triangle *ABC* the points *D* and *E* on *AB* are entered in such a way that the triangles *ABC*, *ACD* and *CBE* are similar to each other.

Therefore: $|AC|:|AB| = |AD|:|AC|$, thus $|AC|^2 = |AB|\cdot|AD|$, and $|BC|:|AB| = |EB|:|BC|$, thus $|BC|^2 = |AB|\cdot|EB|$.

Thus it follows: $|AC|^2 + |BC|^2 = |AB|\cdot(|AD| + |EB|)$.

If the angle at C is a right angle, then the points D and E coincide and it follows $|AC|^2 + |BC|^2 = |AB|^2$.

If the angle at C is obtuse, then the square over the side AB must be reduced by the rectangle of width DE so that it is equal in area to the sum of the squares over the shorter sides: $|AC|^2 + |BC|^2 = |AB|^2 - |AB| \cdot |DE|$.

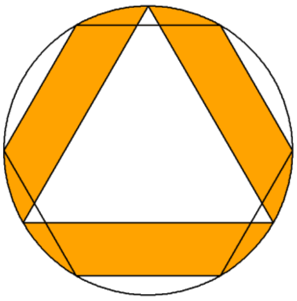
In the case of an acute-angled triangle, it follows accordingly that the square over the side AB must be increased.

He critically examined EUCLID's parallel postulate and tried to prove it (with similar ideas that GIOVANNI GIROLAMO SACCHERI had around the year 1700).

In the preface to *On the Sphere and the Cylinder*, ARCHIMEDES mentioned that he had developed a method to determine the area of a parabolic segment. However, a writing with this content was not known to Islamic scholars until then.

THABIT opened up ARCHIMEDES' lost "integration" methods anew and extended them in principle for arbitrary power functions x^n ($n \in \mathbb{N}$). For fractional exponents he chose a method of exhaustion with a non-equidistant decomposition.

He determined the volumes of solids of revolution, in particular those of domes. For the volume calculation of cylinder, cone and truncated cone he discovered a common rule: $V = \frac{1}{3} \cdot h \cdot \left(A_1 + A_2 + \sqrt{A_1 \cdot A_2} \right)$, where A_1 stands for the area of the base surface and A_2 for that of the top surface of the solid (for the cone we have $A_2 = 0$).



In books on recreational mathematics, one occasionally finds the problem originating from THABIT (on the left) with a surprising result:

- *What proportion of the circular area is coloured*?

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https://www.spektrum.de/wissen/thabit-ibn-qurra-836-901/1151915

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